Charge pumping in one-dimensional Kronig-Penney models

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We consider adiabatic charge transport through one-dimensional open chain for two δ-like pumping sources. We obtain explicitly the charge Q pumped within a period. This charge turns out to be proportional to the conductance G_0. For weak pumping perturbation η, the charge Q is proportional to the area of the contour in parametric space $\eta^2 \sin\phi$, where ϕ is the phase difference of the oscillations of the two parameters. There is an intermediate regime, where Q is proportional to the length of the contour η and to $\sin\phi/2$. For large pumping strength η and not too small ϕ the charge decreases as $(\eta\sin\phi)^{-3}$.

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I. INTRODUCTION

The phenomenon of adiabatic charge pumping, discussed by Thouless,[1] has attracted attention of both experimentalists[2–4] and theorists.[5–11] By pumping we usually mean the dc current (or flow of a fluid) which takes place due to some periodic ac perturbation(s) of the system. Such a dc current is not a persistent current—not an equilibrium response to an external perturbation. Nevertheless it may be entirely adiabatic. It means that the charge transferred within the period of the pumping $t_0=2\pi/\omega$ is independent of this period and remains finite, when the period tends to infinity ($\omega\to 0$). The pumping provides a new way of generating dc currents, which is quite different from the usual application of a dc voltage, and thus can have important practical advantages.

An experimental realization of an adiabatic electron pumping through a quantum dot was reported by Swiktes *et al.*[4] A similar phenomenon, drag of electrons by a traveling acoustic wave in a semiconductor device, was demonstrated by Talyanskii *et al.*[3]

Until now, two approaches to the theoretical description of adiabatic charge transport have been proposed. One of them[7,9] is based on the conventional Green's function formalism. It allows to evaluate explicitly the pumping current for small amplitudes of the pumps and only for weakly disordered systems. The other approach[5] is based on the scattering theory. In spite of these recent developments, many details of the theory, namely the magnitude of the pumped current, its dependence on external tunable parameters (e.g., magnetic field), the relation between ensembles averaged current and its mesoscopic fluctuations, the conditions under which the current is quantized, etc., are not completely understood. There are few theoretical predictions to compare with existing experiments. It is also unclear how this effect changes in the crossover from weak to strong localization.

In this paper we study analytically the pumped current in one-dimensional (1D) Kronig-Penney models, where electrons are subject to a potential that can be represented as a sum of arbitrarily located delta functions with arbitrary weights V_l

$$\tilde{V}(x) = \sum_{l=1}^{N} V_l \delta(x - x_l). \tag{1}$$

Time dependence of the factors V_l can serve as pumping perturbations. Our goal is to evaluate explicitly the scattering matrix elements and their parametric derivatives for a *given* set of the amplitudes by the method of the characteristic determinant.[12] This approach seems to provide a natural model for the study of both weak and strong disorder on the pumping current.

II. PUMPED CURRENT IN TERMS OF THE CHARACTERISTIC DETERMINANT

Consider the pumping charge Q transferred during a single period trough a 1D chain of an arbitrary potential shape $V(x)$ at zero temperature. Let two parameters of the system be modulated periodically. The transferred charge is independent of the frequency ω and, as shown in Ref. 5, is given by

$$Q = \frac{e}{\pi}\int_A \Pi(X,Y)\,dX\,dY, \tag{2}$$

where $\Pi(X,Y)$ is defined as

$$\Pi(X,Y) = \sum_\beta \operatorname{Im}\frac{\partial s_{1\beta}^*}{\partial X}\frac{\partial s_{1\beta}}{\partial Y}, \tag{3}$$

$s_{1\beta}$ ($\beta=1,2$) are the elements of the scattering matrix s and $X=X(t)$ and $Y=Y(t)$ are two external parameters adiabatically varying with time. $\int_A$ denotes integration within the area encompassed by the contour A and the asterisk indicates complex conjugation. According to Eq. (3) $\Pi(X,Y)$ has a meaning of adiabatic curvature.

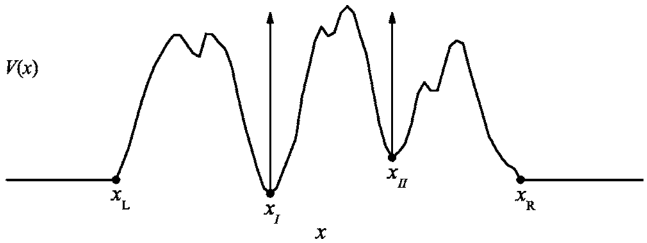

FIG. 1. General potential between x_L and x_R with two varying delta functions at x_I and x_{II}.

To relate the pumped charge Q to the scattering matrix elements $s_{\alpha\beta}$ themselves (not to their derivatives) we use the Fisher-Lee relation[13] between the scattering matrix and the Green's function ($\hbar=1$, $2m_0=1$, and $k=\sqrt{E}$)

$$s_{\alpha\beta}=-\delta_{\alpha\beta}+2ik\ G(x_\alpha,x_\beta). \tag{4}$$

It is well known that the functional derivative of the Green's function $\delta G/\delta V(x)$ can be written as the product of two Green's functions[14]

$$\frac{\delta G(x_\alpha,x_\beta)}{\delta V(x_j)}=G(x_\alpha,x_j)G(x_j,x_\beta). \tag{5}$$

With the help of Eqs. (4) and (5), one can express the adiabatic curvature equation (3) fully in terms of the Green's function

$$\Pi(X,Y)=4k^2\ \mathrm{Im}\left\{G^*(x_1,X)G(x_1,Y)\sum_\beta G^*(X,x_\beta)G(Y,x_\beta)\right\}. \tag{6}$$

As we will see below, for certain cases Eq. (6) can be expressed only through the elements of the scattering matrix s. Now we consider a system where the scattering potential $V(x)$ vanishes outside certain interval: $V(x)=0$ for $x<x_L$ and for $x<x_R$. Let the pumping perturbations be δ-like potentials

$$V_p(x,t)=2kX(t)\delta(x-x_I)+2kY(t)\delta(x-x_{II}), \tag{7}$$

and the amplitudes X, Y demonstrate periodic time evolution with the same period $t_0=2\pi/\omega$. It is convenient to divide the interval $x_L\leqslant x\leqslant x_R$ into three blocks at points $x=x_I$ and $x=x_{II}$ ($x_L\leqslant x_I<x_{II}\leqslant x_R$). The left block is the interval between x_L and x_I, the middle block is between x_I and x_{II}, and the right block is between x_{II} and x_R (see Fig. 1). Using the relation $G(x_\alpha,x)G(x,x_\beta)=G(x_\alpha,x_\beta)G(x,x)$, valid for $x_\alpha<x<x_\beta$ (see, e.g., Ref. 15), one can rewrite Eq. (6) as

$$\Pi(X,Y)=\frac{1}{4k^2}\ \mathrm{Im}[(t_{LI}^*t_{LII})^2+T(r_{LI}^*+r'_{LII}+r'_{LII}r_{LI}^*)]. \tag{8}$$

Here t_{LI} is the amplitude of transmission through the left block for an electron incident from the left. t_{LII} is the amplitude of transmission through the left and middle blocks combined. r_{LI} is the reflection amplitude off the same two blocks (the electron is incident from the left). And finally, the r'_{LII} is the amplitude of reflection off the middle and right blocks combined for an electron incident from the right. $T=tt^*$ is the coefficient of transmission through the whole system.

Below, we concentrate on the simplest case where $x_I=x_L$ and $x_{II}=x_R$, and thus simplify Eq. (8)

$$\Pi(X,Y)=\frac{1}{4k^2}\mathrm{Im}\left(t^2+r^*t^2+r^*T\right). \tag{9}$$

To arrive at Eq. (9) we have used the time-reversal symmetry for t and the current-conservation requirement,[16] which implies $tr^*+r't^*=0$ with r and r' being the reflection amplitudes from the left and the right of the scatter, respectively.

For concreteness, we will evaluate t and r for a disordered Kronig-Penney model:

$$V(x)=\sum_{l=2}^{N-1}V_l\delta(x-x_l). \tag{10}$$

The Hamiltonian of the particle is

$$H=k^2+V(x)+V_p(x,t), \tag{11}$$

where $V_p(x,t)$ is determined by Eq. (7) with $x_I=x_1$ and $x_{II}=x_N$.

We can express the reflection and transmission amplitudes, r and t, for the potential $V(x)$ from its characteristic determinant D_N, introduced in Ref. 12:

$$D_N=\det M_{n,l}^{(N)}, \tag{12}$$

where

$$M_{n,l}^{(N)}=\delta_{nl}+\frac{iV_l}{2k}\exp\{ik|x_l-x_n|\},\quad 1\leqslant n,l\leqslant N. \tag{13}$$

The characteristic determinant D_N can be presented as a determinant of a tridiagonal matrix that satisfies the following recurrence relationship:

$$D_N=A_ND_{N-1}-B_ND_{N-2}, \tag{14}$$

where D_{N-1} (D_{N-2}) is the determinant equation (13) with the Nth [and also the $(N-1)$th] row and column omitted.

$$D_{N'}=\det M_{n,l}^{(N')},\quad 1\leqslant n,l\leqslant N', \tag{15}$$

and $M_{n,l}^{(N')}$ is still determined by Eq. (13).

The coefficients A_N, B_N can be obtained from the explicit form of $M_{n,l}^{(N)}$. For $N>1$ we have

$$A_N=1+B_N+\frac{iV_N}{2k}[1-e^{2ik(x_N-x_{N-1})}], \tag{16}$$

and

$$B_N=\frac{iV_N}{V_{N-1}}e^{2ik(x_N-x_{N-1})}. \tag{17}$$

The initial conditions for the recurrence relations are

$$D_0=1,\quad D_{-1}=0,\quad A_1=1+\frac{iV_1}{2k}. \tag{18}$$

The transmission amplitude t is the inverse of the characteristic determinant D_N multiplied by the phase accumulated during the transmission, i.e.,

$$t = e^{ik(x_N - x_1)} D_N^{-1}, \tag{19}$$

while the reflection amplitude r is

$$r = -\frac{1}{iX}\frac{D_N - D_{-1+N}}{D_N} - 1 \equiv i\frac{\partial \ln t}{\partial X} - 1, \tag{20}$$

where D_{-1+N} is the characteristic determinant without the first delta function (i.e., $X=0$).

The determinant D_N has a well-defined continuous limit when the potential $V(x)$ is an arbitrary smooth function of coordinate x. It is nothing but the inverse transmission amplitude $D=t^{-1}$. Let $V(x)=0$ for $x>x_0$. One can determine D as

$$D = \lim_{y \to x_0} D(y). \tag{21}$$

Here $D(y)$ is the inverse amplitude $t(y)$ of the transmission from the left through the potential

$$V_y(x) = V(x)\Theta(y - x), \tag{22}$$

where $\Theta(z)=0$ for $z<0$ and $\Theta(z)=1$ for positive z. For the complex reflection amplitude $r(y)$ and the inverse transmission amplitude $t^{-1}(y)=D(y)$ can be calculated using two first-order differential equations with appropriate initial conditions[17]

$$\frac{dr(y)}{dy} = -\frac{1}{2ik}V(y)[e^{iky} + r(y)e^{-iky}]^2, \tag{23}$$

$$\frac{dD(y)}{dy} = \frac{1}{2ik}D(y)V(y)[1 + r(y)e^{-2iky}]. \tag{24}$$

From these two equations can be obtain a second-order differential equation for $D(y)$

$$\frac{d^2D(y)}{dy^2} + \left(2ik - \frac{d \ln V(y)}{dy}\right)\frac{dD(y)}{dy} - V(y)D(y) = 0. \tag{25}$$

III. CALCULATIONS

In the absence of the pumping perturbations [in Eq. (11) $V_p=0$] the elements of the scattering matrix

$$s_0(E) = \begin{pmatrix} r_{0L} & t_0 \\ t_0 & r_{0R} \end{pmatrix} \tag{26}$$

can be written as

$$t_0 = \sqrt{G_0}e^{i\varphi_0}, \tag{27}$$

$$r_{0L,R} = \sqrt{1 - G_0}e^{i(\varphi_0 \pm \varphi_a)}, \tag{28}$$

where G_0 is the conductance of the block with $N-2$ delta potentials. φ_0 is the phase of the transmission amplitude and φ_a is the phase difference between the reflection from the left and from the right.

Our goal now is to write explicitly the dependence of the characteristic determinant D_N on X and Y. Applying the recurrence relation for the characteristic determinant, Eq. (14), to both ends of the system we rewrite D_N as

$$D_N(X,Y) = \tilde{D}_0[1 + AiX + BiY + CXY], \tag{29}$$

where $\tilde{D}_0$ is the characteristic determinant for the case when $V_p=0$ (i.e., $X=Y=0$), and A, B, and C are coefficients independent of X and Y that can be expressed through $\tilde{D}_0$. These coefficients are defined as

$$A = 1 - i\sqrt{1 - G_0}\exp\{i[\varphi_0 + \varphi_a + 2k(x_2 - x_1)]\}, \tag{30}$$

$$B = 1 - i\sqrt{1 - G_0}\exp\{i[\varphi_0 - \varphi_a + 2k(x_N - x_{N-1})]\}, \tag{31}$$

$$\begin{aligned} C = {} & 2i\exp\{i[\varphi_0 + k(x_N - x_{N-1} - x_1 + x_2)]\} \\ & \times\{\sin[\varphi_0 + k(x_N - x_{N-1} - x_1 + x_2)] \\ & + \sqrt{1 - G_0}\cos[\varphi_a - k(x_N - x_{N-1} - x_2 + x_1)]\}. \end{aligned} \tag{32}$$

Using Eq. (29) for D_N, and substituting Eq. (19) and Eq. (20) into Eq. (9) we arrive at

$$\Pi(X,Y) = \frac{2\,\mathrm{Im}\,C + |C|^2(X+Y)}{8k^2G_0|D_N(X,Y)|^4}. \tag{33}$$

Keeping in mind the explicit expression for $\Pi(X,Y)$, Eq. (33), the pumped charge Q, given by Eq. (2), will be of the form

$$Q = \frac{e}{2\pi G_0}\int_A \frac{2\,\mathrm{Im}\,C + |C|^2(X+Y)}{|D_N(X,Y)|^4}dXdY. \tag{34}$$

In order to proceed further, let us discuss the case when the quantities A and B [Eqs. (30) and (31)] are equal, i.e., $\varphi_a=k(x_N-x_{N-1}-x_2+x_1)$ and the two parameters X and Y are varied periodically,

$$X = X_0 + \eta\cos\omega t \tag{35}$$

and

$$Y = X_0 + \eta\cos(\omega t - \phi), \tag{36}$$

where X_0 is the initial potential and ϕ is the phase difference of the two parameters. In this case, we can rewrite the characteristic determinant as

$$D_N = \tilde{D}_0[C\eta^2\cos^2(\omega t - \phi/2) + 2iK\eta\cos(\omega t - \phi/2) + L], \tag{37}$$

where

$$K = (A - iCX_0)\cos\phi/2, \tag{38}$$

and

$$L = 1 + 2iAX_0 + CX_0^2 - C\eta^2\sin^2\phi/2. \tag{39}$$

The transmission coefficient is given by

$$T_N^2 \equiv |D_N(\omega t,\phi,\eta)|^{-4} = \frac{G_0^2}{8|\Delta|^2} \operatorname{Re} \sum_{j=1}^{2} \alpha_j \left[\frac{1}{[\eta\cos(\omega t-\phi/2)-c_j]^2} + \frac{2\beta_j}{\eta\cos(\omega t-\phi/2)-c_j} \right], \quad (40)$$

with the different constants c_j, α_j, β_j defined as

$$c_{1,2} = -\frac{i}{C}(K \pm \Delta^{1/2}), \quad (41)$$

$$\alpha_{1,2} = \left(\frac{1}{c_1^* - c_{1,2}} - \frac{1}{c_2^* - c_{1,2}} \right)^2, \quad (42)$$

$$\beta_{1,2} = \frac{1}{c_1^* - c_{1,2}} + \frac{1}{c_2^* - c_{1,2}} \pm \frac{1}{c_2 - c_1}, \quad (43)$$

and

$$\Delta \equiv K^2 + LC = (A - iCX_0)^2 \cos^2\frac{\phi}{2} + \left(1 + 2iAX_0 + CX_0^2 - C\eta^2 \sin^2\frac{\phi}{2}\right)C. \quad (44)$$

With the specific form chosen for the perturbing potentials, the pumped current is given by an integral over the amplitude and the period t_0 of the perturbations

$$Q = \frac{e\omega\sin\phi}{\pi G_0}\int_0^{\eta} \eta' d\eta' \int_0^{t_0} T_N^2(\omega t,\phi,\eta') \times \left\{ \operatorname{Im} C + |C|^2 \left[X_0 + \eta' \cos\frac{\phi}{2}\cos\left(\omega t - \frac{\phi}{2}\right) \right] \right\} dt. \quad (45)$$

The integral over the period in Eq. (45) can be performed in the complex plane, and we finally arrive at

$$Q = \frac{eG_0 \sin\phi}{4}\int_0^{\eta} \frac{\eta' d\eta'}{|\Delta|^2} \operatorname{Re}\sum_{j=1}^{2} \frac{\alpha_j}{\gamma_j 3}\left[(\operatorname{Im} C + |C|^2 X_0) \times (c_j - 2\beta_j\gamma_j^2) + |C|^2(\eta'^2 + 2\beta_j\gamma_j^3 - c_j\gamma_j^2)\cos\frac{\phi}{2} \right]. \quad (46)$$

Here

$$\gamma_j = \sqrt{c_j^2 - \eta'^2}\,\operatorname{sgn}(|c_j + \sqrt{c_j^2 - \eta'^2}| - 1) \quad (47)$$

and always has positive imaginary part $\sqrt{c_j^2 - \eta'^2}$.

This is our main general result. In the following section we analyze its limits.

IV. RESULTS

Let us start our discussion of the results from the weak-pumping regime, which corresponds to the condition

$$\eta^2 \sin^2\phi/2 \ll \max|1/C, 2X_0A/C, X_0^2|. \quad (48)$$

In this limit, Δ, and consequently $c_{1,2}$, do not depend on η' and for the integral equation (46) we have

$$Q \approx \frac{eG_0\sin\phi}{4|\Delta|^2} \operatorname{Re}\sum_{j=1}^{2}\alpha_j \left\{ (\sqrt{c_j^2 - \eta^2} - c_j) \times [\Gamma(2\beta_j - \sqrt{c_j^2 - \eta^2}^{-1/2}) + |C|^2 X_0] + \eta^2 \beta_j |C|^2 \cos\frac{\phi}{2} \right\}, \quad (49)$$

where $\Gamma = \operatorname{Im} C + |C|^2(X_0 + \cos\phi/2)$. Equation (49) can be further simplified if, besides the inequality, Eq. (48), the condition

$$|1 + 2iX_0A + CX_0^2| \ll |A/C - iX_0|^2 \cos^2\phi/2 \quad (50)$$

is also satisfied. Then Q reads

$$Q \approx \frac{eG_0(\operatorname{Im} C + |C|^2 X_0)\sin\phi}{\sqrt{|1 + 2iX_0A + CX_0^2|^2 + 4\eta^2\cos^2\phi/2}}\eta^2. \quad (51)$$

As follows from Eq. (51), if the pumping perturbation η is weak the charge Q is proportional to the area of the contour in parametric space[5,7] and to $\sin\phi$. With increasing pumping strength, Q becomes proportional to the length of the contour[9] and to $\sin\phi/2$, where ϕ is the phase difference of the oscillations of the two parameters X and Y.

Consider now strong pumping

$$\eta^2 \sin^2\phi/2 \gg \max|1/C, 2X_0A/C, X_0^2| \quad (52)$$

and

$$\eta^2 \tan^2\phi/2 \gg |A/C - iX_0|^2. \quad (53)$$

Under these conditions the pumped charge becomes approximately equal to

$$Q \approx \frac{2eG_0(\operatorname{Im} C + |C|^2 X_0)}{3|A - iCX_0|^3|C|}\frac{1}{\eta^3\sin^3\phi}, \quad (54)$$

i.e., for large pumping strength η the charge behaves as η^{-3}.

To illustrate these results we present numerical calculations of Eq. (46) for the case of only two delta functions and $\phi = \pi/2$. We expect the same behavior of Q as long as $G_0 = 1$. Let us start with the case $X_0 = 0$. The pumped charge dependence on the amplitude of oscillation is presented in Fig. 2 for an incident energy $E = k^2$ given by $ka = 3.1$ (a is the distance between the two delta potentials). The results for other incident energies are similar if we plot $Q/\cos(ka)$ (rather than Q) as a function of $\eta\sin(ka)$. In the inset of Fig. 2, we represent the large η limit of the absolute value of the same data as in the main part of the figure on a double logarithmic scale. The data fit fairly well a straight line with slope -3.18. One can see that in the strong pumping regime the charge indeed tends to zero as η^{-3}, within numerical uncertainties, in agreement with Eq. (54).

If the strengths of the oscillating delta potentials contain a constant term X_0, the behavior of the charge is more interesting as can be seen in Fig. 3, where the $Q(\eta)$-dependence at $X_0 = 100$ is presented. In contrast with the $X_0 = 0$ case, the

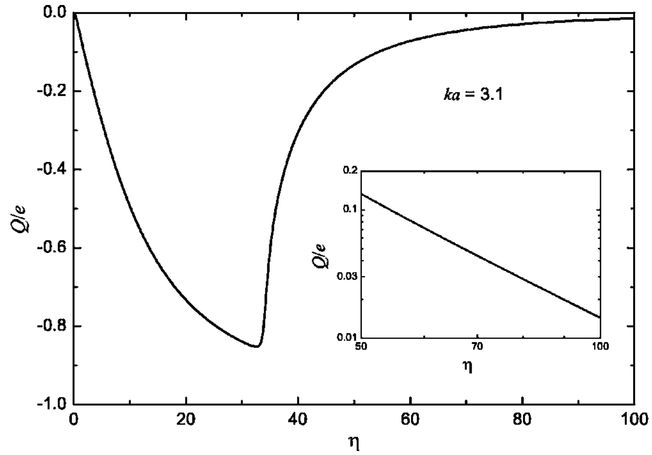

FIG. 2. Pumped charge in units of e as a function of the amplitude of oscillation of the intensity of the two delta potentials without a constant term, $X_0=0$. We have considered an incident energy given by $ka=3.1$ (a is the distance between the two delta potentials). Inset: large η limit of the same data in absolute value as in the main part of the figure on a double logarithmic scale.

pumped charge approaches the unit of charge in a finite interval of oscillating strengths.

The pumped charge, in the region where it is almost quantized, gets closer and closer to the unit charge as we increase X_0, for a constant incident energy. In Fig. 4, we represent the difference between the maximum value of the pumped current and the unit of charge as a function of X_0 on a double logarithmic scale for the incident energy that corresponds to $ka=3.1$. The slope of the fitted straight line is -1.940. Within numerical errors, the pumped charge approaches e as X_0^{-2}. In other words in the limit of big X_0, $Q \approx 1-G$ according to Ref. 9, where $G \approx X_0^{-2}$ is the conductance of the system that tends to zero when X_0 increases.

V. CONCLUSION

We evaluated explicitly adiabatic charge transport through one-dimensional open chain of arbitrarily located delta functions with arbitrary amplitudes. Two of these functions are supposed to oscillate in strength and thus act as pumping sources. We show that the charge is proportional to the conductance G_0 of the system without these two sources. If the pumping perturbation η is weak, the charge Q is proportional to the area of the contour in parametric space.[5,7] With increasing pumping strength, Q becomes proportional to the length of the contour[9] and to $\sin \phi/2$, where ϕ is the phase difference of the oscillations of the two parameters X and Y. For even larger pumping strength η the charge decreases as $(\eta \sin \phi)^{-3}$. In the case when the conductance of the system is close to its maximum $G_0 \equiv 1$, in a finite range of pumping potentials the charge is almost quantized, gets closer and closer to the unit charge as we increase X_0, the constant term of the oscillating potentials. The difference between the pumped current and the unit of charge tend to zero as $G \approx X_0^{-2}$. In other words in the limit of big X_0, $Q \approx 1-G$, where $G \approx X_0^{-2}$ is the conductance of the system that tends to zero when X_0 increases.

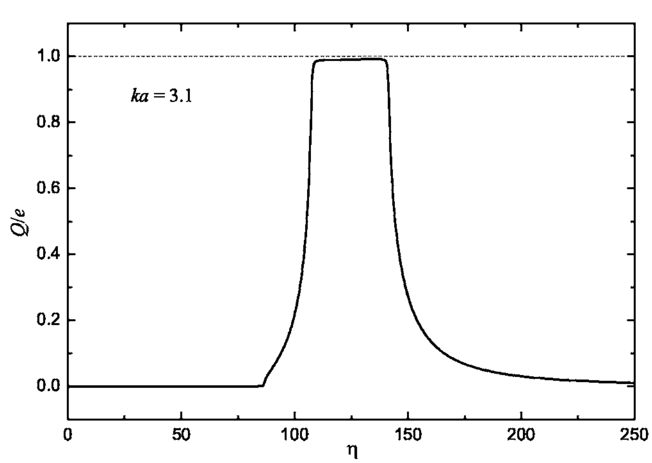

FIG. 3. Pumped charge in units of e as a function of the amplitude of oscillation of the intensity of the two delta potentials. These intensities have a constant term $X_0=100$ and we have considered an incident energy given by $ka=3.1$.

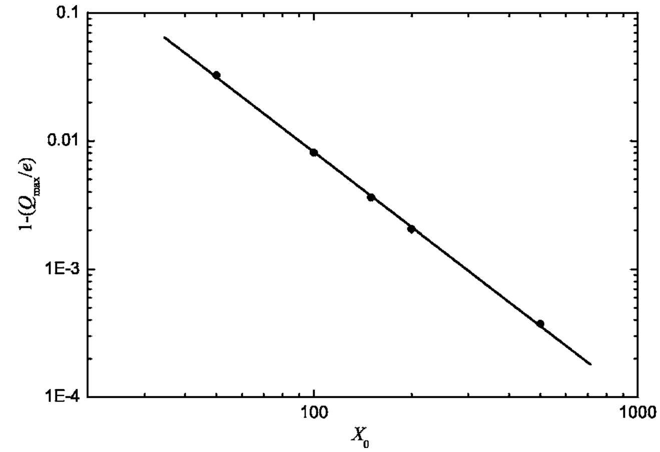

FIG. 4. Difference between the pumped current and the unit of charge as a function of X_0 on a double logarithmic scale. The incident energy is given by $ka=3.1$. The slope of the fitted straight line is -1.940.

We believe that a similar analysis can be performed for the case of a smooth random potential. One has just to employ the second order differential equation Eq. (25) instead of the recursion relation Eq. (14).

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APPENDIX: ADIABATIC CURVATURE $\Pi_\alpha(X,Y)$

Let us define $\Pi_\alpha(X,Y)$ which characterizes the direction of the incident flow

$$\Pi_\alpha(X,Y) = \sum_\beta \frac{\partial s^*_{\alpha\beta}}{\partial X}\frac{\partial s_{\alpha\beta}}{\partial Y}, \tag{A1}$$

where $\alpha=L,R$ and $s_{\alpha\beta}$ are the elements of the scattering matrix s. Note that $\Pi_\alpha(X,Y)$ defined by Eq. (3) is nothing but

$\mathrm{Im}\,\Pi_L(X,Y)$. The matrix elements $s_{\alpha,\beta}$ can be expressed through the reflection R and transmission T coefficients ($T+R=1$), and two phases: ψ for the transmission of the amplitude and ψ_α, which characterizes the asymmetry between the reflection to the left and to the right from the 1D potential $V(x)$:

$$s(E) = \begin{pmatrix} r & t \\ t & r' \end{pmatrix} = e^{i\psi}\begin{pmatrix} -i\sqrt{R}e^{i\psi_a} & \sqrt{T} \\ \sqrt{T} & -i\sqrt{R}e^{-i\psi_a} \end{pmatrix}. \quad \text{(A2)}$$

We assume that the quantities R, T, ψ, and ψ_a are functions of the external parameters X and Y.

As it was shown in the text the adiabatic curvature can be presented in the form

$$\Pi_L(X,Y) = \frac{(t^2+T)(r^*+1)}{4k^2} \quad \text{(A3)}$$

provided that $x_I=x_L$ and $x_{II}=x_R$ and electrons are incident from the left. Analogously for the right flow the adiabatic curvature is

$$\Pi_R(X,Y) = \frac{(t^{*2}+T)(r'+1)}{4k^2} = \frac{(t^*+t)^2}{4k^2} - \Pi_L(X,Y). \quad \text{(A4)}$$

From Eqs. (A3) and (A4) follows the general relation

$$\mathrm{Im}\,\Pi_L(X,Y) = -\ \mathrm{Im}\,\Pi_R(X,Y). \quad \text{(A5)}$$

The real parts of $\Pi_L(X,Y)$ and $\Pi_R(X,Y)$ characterize the contribution of interference effects to the heat production and to the noise.[18] Contrary to Eq. (A5) there is no general relation between the real parts, i.e., $\mathrm{Re}\,\Pi_L(X,Y)\neq \mathrm{Re}\,\Pi_R(X,Y)$. Indeed, writing the scattering matrix in the form Eq. (A2) we obtain

$$\mathrm{Im}\,\Pi_L(X,Y) = -\ \mathrm{Im}\,\Pi_R(X,Y) = \frac{T}{2k^2}\cos\psi(\sin\psi + \sqrt{R}\cos\psi_a), \quad \text{(A6)}$$

$$\mathrm{Re}\,\Pi_L(X,Y) = \frac{T}{2k^2}\cos\psi(\cos\psi + \sqrt{R}\sin\psi_a), \quad \text{(A7)}$$

$$\mathrm{Re}\,\Pi_R(X,Y) = \frac{T}{2k^2}\cos\psi(\cos\psi - \sqrt{R}\sin\psi_a). \quad \text{(A8)}$$

So only when the phase asymmetry is $\psi_a=0$, i.e., for a spatially symmetric barrier and for symmetrically located x_L and x_R, real parts of Π_L and Π_R are related

$$\mathrm{Re}\,\Pi_L(X,Y) = \mathrm{Re}\,\Pi_R(X,Y). \quad \text{(A9)}$$

Note that $\Pi_L(X,Y)=\Pi_R(X,Y)\equiv 0$ and thus pumping is absent when $\psi=\pi/2$. This disappearance of the pumping is not a miracle: condition $\psi=\pi/2$ is equivalent to $Y=mX+b$ with constant m and b.

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